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Enhanced Adaptive Neural-Fuzzy Inference System for Dynamic Time Series Prediction Using Self-Feedback and Hybrid Training

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Highlights

- Enhanced ANFIS for chaotic time series forecasting.
- Incorporates self-feedback relationships for dynamic systems.
- Utilizes hybrid ICA-LSE approach for training.
- Outperforms previous methods in prediction accuracy.

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Abstract

Predicting time series, especially those originating from chaotic and nonlinear dynamic systems, is a critical research area with broad applications across various fields. Neural networks and fuzzy systems have emerged as leading methods for forecasting chaotic time series. This study introduces an improved adaptive neural-fuzzy inference system (ANFIS) specifically tailored for forecasting chaotic time series. Unlike traditional ANFIS models, which are primarily designed for static problems, this enhanced version incorporates self-feedback relationships from previous outputs to capture the time dependencies inherent in dynamic systems. Additionally, a hybrid approach combining the Imperialist Competitive Optimization Algorithm (ICA) and Least Squares Estimation (LSE) is employed to train the neural-fuzzy system and update its parameters. This method circumvents challenges associated with training gradient-based algorithms. The proposed technique is applied to predict and model multiple nonlinear and chaotic time series from real-world scenarios. Comparative analyses with recent works demonstrate the superior performance of the proposed method, particularly in terms of the prediction total error criterion for time series modeling and forecasting. These results highlight the effectiveness of incorporating self-feedback relationships and utilizing the CCA-LSE hybrid approach in enhancing the predictive capabilities of adaptive neural-fuzzy inference systems for chaotic time series.

1. Introduction

Most of the phenomena of mechanical systems can be modeled using appropriate differential equations, which need to be solved in order to study their behavior[1], [2], [3], [4], [5], [6]. Time series forecasting holds a prominent position among forecasting disciplines, involving the collection and analysis of past observations of a variable to establish key relationships and formulate a descriptive model[7], [8], [9]. Then, the resulting model has been used to extrapolate time series in the future. Forecasting time series is an essential topic with broad applications in science, engineering, medicine, economics, etc. In general, a time series has properties such as non-linearity, chaos, non-stationarity, and periodicity, such as seasonality[10], [11], [12]. Between the different kinds of time series, chaotic data is usually found in natural occurrences. Predicting the chaotic time series of dynamic systems is a novel examination topic that has attracted the attention and efforts of many scientists[13], [14], [15].

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Neural networks [16], [17], [18] especially fuzzy neural networks (FNNs) are one of the proposed structures for predicting time series[19]. Among these networks, the ANFIS adaptive neural-fuzzy inference system is an adaptive neural network based on fuzzy inference, which is trained by optimizing the parameters of the front part. There is an inherent problem in the adaptive neuro-fuzzy inference system[20]. This problem is driven by its progressive nature, which limits its ability to model static problems. Because of this, it cannot be used successfully in dynamic problems such as time series forecasting. In the field of time series, many statistical and classical methods have been presented. However, these methods are often complicated and perform poorly in the face of large and chaotic data[21]. In recent years, investigators have used many forecasting approaches such as the wavelet method[22], neural networks[23], [24], fuzzy systems[25], neuro-fuzzy[26], and evolutionary algorithms [27] have emerged. In recent years, many works related to recurrent neural networks have been presented to solve these problems with the dynamics of absorbing and storing information[28]. However, the training of recurrent neural network, due to its complex network structure, is more difficult and requires more calculations compared to forward networks. In addition, it has been proven that in the structure of progressive networks, the efficiency of fuzzy neural networks is better than that of neural networks[29]. To overwhelmed this structural problem in feed-forward neural-fuzzy networks, researchers tried to build recurrent neural-fuzzy networks with the formation of internal feedback and time delay and used them to control, identify, and predict systems[30]. Previously, similar works and research have been carried out for prediction and identification using wavelet-based recurrent fuzzy neural network (WRFNN)[31], as well as a self-feedback adaptive neuro-fuzzy inference system with a local batch search algorithm[32].

In recent years, predicting chaotic and nonlinear time series data has emerged as a critical research area with widespread applications across various fields, including finance, environmental science, and engineering. However, the inherent complexities of dynamic systems pose significant challenges to accurate forecasting, necessitating the development of more sophisticated modeling techniques. Traditional forecasting methods, such as linear regression or autoregressive models, often struggle to capture the intricate dynamics and nonlinear behaviors exhibited by chaotic time series data. Consequently, there exists a pressing need for advanced forecasting methodologies capable of effectively handling the complexities of dynamic time series data. In this context, this study introduces an enhanced adaptive neural-fuzzy inference system (ANFIS) tailored specifically for forecasting chaotic time series data. By addressing the limitations of traditional ANFIS models and leveraging self-feedback mechanisms, this novel approach aims to enhance the predictive capabilities of ANFIS in capturing the complex temporal dependencies inherent in dynamic systems. Through this research, we seek to contribute to the advancement of time series prediction methodologies and address the pressing need for more effective forecasting techniques in the domain of chaotic and nonlinear time series analysis.

The study proposes a method to enhance the Adaptive Neural-Fuzzy Inference System (ANFIS) for dynamic problems, addressing its inherent limitations. A self-feedback mechanism is introduced to overcome memory limitations and establish temporal relationships between inputs and outputs. The method integrates Independent Component Analysis (ICA) and Least Squares Estimation (LSE) for parameter adaptation, offering advantages over gradient-based methods. By combining learning techniques and self-feedback, the proposed approach improves ANFIS performance in time series forecasting. Simulation results on chaotic time series datasets (Mackey-Glass, Sunspot Number) demonstrate effectiveness compared to established methods, evaluated using Haykin's criteria for forecasting accuracy. This novel approach represents a significant advancement in ANFIS application for dynamic systems forecasting. The innovation lies in the combination of a self-feedback mechanism with Independent Component Analysis (ICA) and Least Squares Estimation (LSE) to enhance the Adaptive Neural-Fuzzy Inference System (ANFIS) for time series forecasting. This integration allows for overcoming inherent limitations of ANFIS, such as memory constraints and difficulties in parameter adaptation.

2. Basic concepts

2.1. Imperialist competitive algorithm (ICA)

ICA is a new evolutionary search procedure based on socio-political evolution. Considering the phenomenon of colonization as an inseparable part of the course of human historical evolution, this algorithm is used as a source of inspiration to create an efficient and new algorithm in the field of evolutionary calculations. Figure (1) illustrates the steps of the ICA method.

Comparable to other evolutionary procedures, ICA starts with a random initial population and an objective function that is calculated for all of them. The strongest countries are chosen as the colonizers, and the weaker countries are selected as the colonies of these colonizers.

Then, there was competition between the colonists to get more colonies. The best colonizer has more chances to have more colonies. Then, each colonizer forms an empire with his colonies. Figure 1 shows the different steps of the ICA algorithm. More details of this optimization algorithm are provided in the reference.

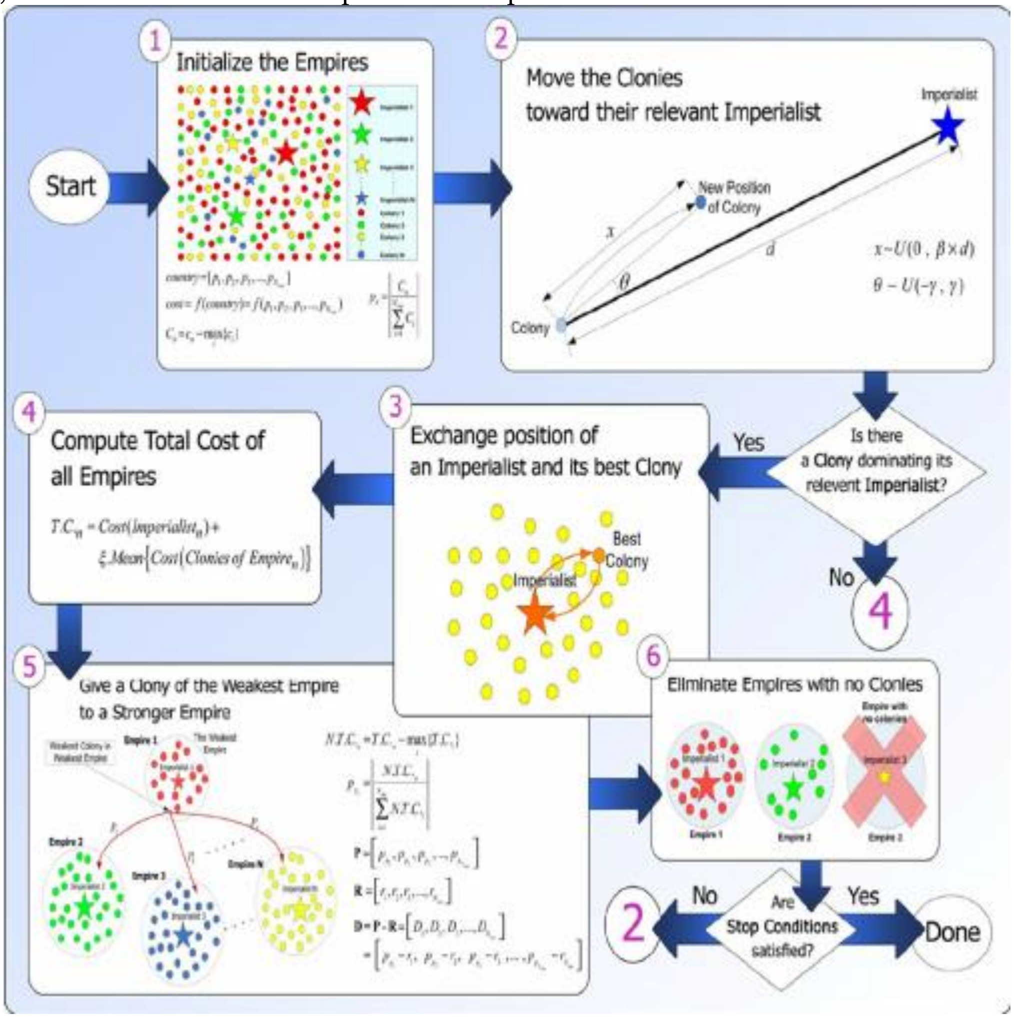

Fig 1 Overview of the Imperialist competitive algorithm [33]

2.2. Adaptive neural-fuzzy inference system (ANFIS)

Both neural networks and fuzzy structures [33] exhibit inherent capabilities to manage uncertainty and noise. Adaptive fuzzy-neural networks, akin to fuzzy systems, are structured into two components. The initial part constitutes the front section, while the subsequent part forms the tail section, interconnected in a network format through rules. Figure 2 illustrates the configuration of a type III of ANFIS with two inputs. The system's architecture encompasses a total of 5 layers [34]. Type III ANFIS is a variation of the ANFIS architecture that incorporates additional layers and functionality compared to the original Type I and Type II ANFIS models. In Type III ANFIS, the structure typically involves more complex neural network architectures, potentially including recurrent neural networks (RNNs) or other forms of feedback connections. These additional layers and connections allow for more sophisticated modeling of temporal dependencies and dynamic behaviors in time series data.

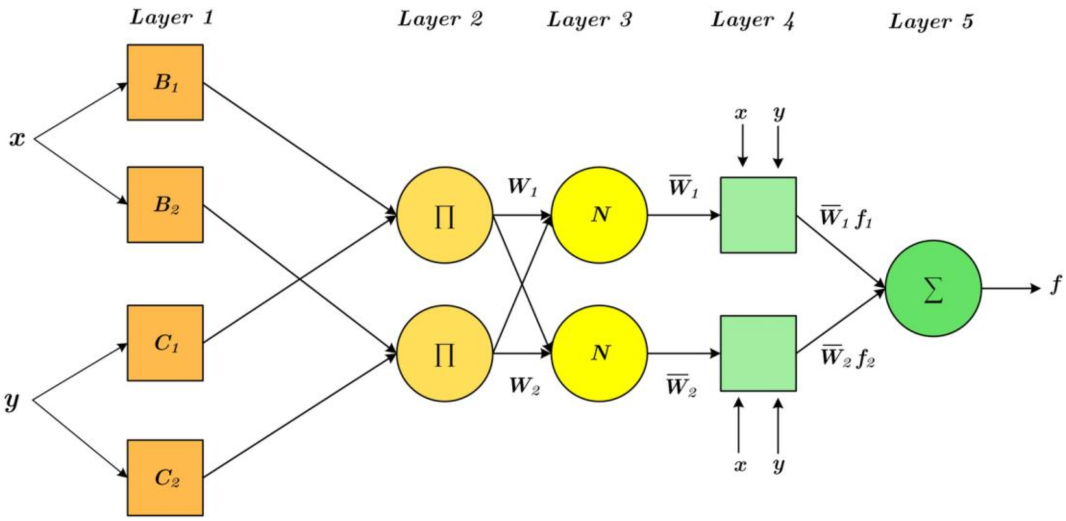

Fig. 2 The configuration of a type III of ANFIS featuring two inputs and a single output [34]

The 1st layer implements the fuzzification process. In this layer, each node shows a membership function, which is the learnable parameter of the front part. In the second layer, each firing strength is calculated. In the third layer, the firepower of each rule is normalized according to the firepower of other laws. In the fourth layer, the output of each rule is obtained, and finally, the last layer calculates the output of the fuzzy system by adding the outputs of the fourth layer.

Figure 2 shows two labels for each input. According to Type III, the rules are as a:

$$w_i = \mu_{A_i}(x) + \mu_{B_i}(x).\, i = 1.2 \quad (1)$$

$$\bar{w}_i = \frac{w_1}{w_1 + w_2}.\, i = 1.2 \quad (2)$$

$$f_1 = b_1 y + a_1 x + c_1 \quad (3)$$

$$f_2 = b_2 y + a_2 x + c_2$$

$$f = \frac{w_1 f_1 + w_2 f_2}{w_1 + w_2 f} = \bar{w}_1 f_1 + \bar{w}_2 f \quad (4)$$

The adaptive neural-fuzzy inference system exhibits a noteworthy capability in approximation, relying on its proficiency in partitioning the input space through the determination of membership functions in the front section for each input. In this context, Gaussian membership functions are employed. The definition of these functions is as follows:

$$\mu_{A_i}(x) = exp\left(-\frac{1}{2}\left[\frac{x - m_i}{\sigma_i}\right]\right) \quad (5)$$

2.2.1. *Self-feedback ANFIS*

In the Self-feedback ANFIS model, the dynamic characteristic originates from the feedback loop involving the system's own output in preceding steps. This feedback mechanism enables the system to preserve a memory of its previous states, incorporating information from both prior and current states to calculate new values. From a scientific perspective, this embedded time delay shares similarities with self-feedback, leading to varied dynamic behaviors. These behaviors are instrumental in augmenting the system's ability to provide predictions with greater precision and accuracy.

To tackle challenges inherent in dynamic problems related to the adaptive neural-fuzzy inference system, a solution involves reintroducing the output of the progressive network to the system's inputs, as illustrated in Figure 3. Utilizing output feedback from previous stages, the output at the moment "t" is defined as a function that incorporates both the current and preceding inputs and outputs, expressed through the following relationship[35]:

$$y(t) = f[R(t-1).,,,.R(t-M).R(t-1)] \quad (6)$$

where (R (t -1),...,R (t - M) remain direct inputs and y(t -1),..., y(t - N) stand feedback inputs given from the output at different times.

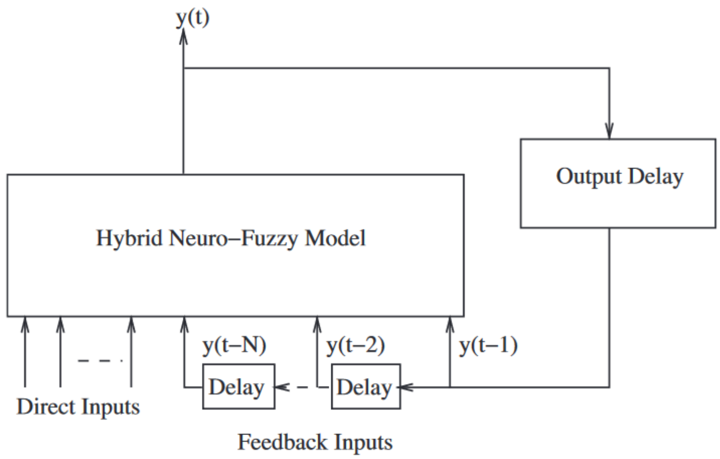

Fig. 3 Structure of the ANFIS with output feedback

2.2.2. Training neural-fuzzy inference system

Following the creation of the ANFIS, techniques for training its parameters were introduced. For example, in Ref.[36], the integration method of the Min-Max and ANFIS model is proposed to determine the neural-fuzzy network and determine the set of optimal rules of the fuzzy system. Jang and Mizutani [37] proposed an application of the Levenberg–Marquardt method, which is basically a nonlinear least square method. In another article, Jang [38] presented a scheme for input selection to learn the adaptive neural-fuzzy inference system. In Ref.[39], Jang presented four methods for updating system parameters: 1) gradient descent: all parameters are trained with the help of gradient descent; 2) gradient descent and a LSE step are used in the first step to initialize the parameters of the next section; 3) gradient descent and LSE; 4) Sequential LSE: This method linearizes all the parameters and uses the developed Kalman filter algorithm to update the parameters. The methods proposed so far depend on the gradient and are still applied due to their suitable performance. The methods that depended on the least squares are effective methods for optimizing the parameters. Training is heavily reliant on the parameters of the front part, and conventional methods are deemed unsuitable due to the intricate nature of gradient calculations and the nonlinear involvement of front parameters in the output. Population-based approaches, such as genetic algorithms, group optimization of particles, ant colonies, and other related methods, are proposed to solve this problem. Many works for training neural-fuzzy networks with a combination of gradient descent, evolutionary algorithms and least square, and Kalman filter algorithm methods have been proposed.

In this work, a method is presented for training the constraints of the ANFIS, which has less complexity and more accurateness, and in which the training of the parameters of the front part and the tail part is performed as an iterative procedure with the evolutionary algorithm of ICA and LSE. The next section presents the enhanced adaptive neural-fuzzy inference system for prediction.

3. Proposed method

In this section, the algorithm of the proposed method for forecasting time series is described step by step.

First step: Reconstruction of the state space of chaotic time series. In order to study the geometric and dynamic properties of a certain system, the state space description is utilized. One of the most fundamental steps in the analysis of time series resulting from a nonlinear process is the reconstruction of the state space with limited dimensions using these series so that it is equivalent to the state space of the data-generating process. With the inference theory, the problem of state space reconstruction from time series is solved. In essence, the points on the absorber of the system exhibit a one-to-one correspondence with measurements taken of the system's dynamic variables. Additionally, these points encapsulate comprehensive information about the current state of the system. The one-to-one relationship implies that the phase space states are precisely associated with the measurements. Consequently, the quest is for a mapping from the system absorber to the reconstructed space, ensuring that this mapping is both one-to-one and preserves the vital information of the system. This conceptual definition underscores the importance of a mapping that accurately captures and retains the system's information. By applying Tokens theory in reconstructing the state space from a chaotic time series, the system's state space is reconstructed with the help of two parameters, the inference dimension D and the delay time τ. If the time series is in the form [q(1), q(2),..., q(N)], an embedded phase vector u(i) is as follows:

$$u(i) = [q(i).q(i-\tau).,,,.q(i-(D-1)\tau)] \quad (7)$$

where D remains the inference dimension, τ stays the time delay, and $u(i)$ is the D-dimensional phase space. Hence, the input matrix is defined as Eq. (8).

$$U = \begin{bmatrix} q(i) & q(i+1) & \cdots & q(i+n) \\ q(i-\tau) & q(i+1-\tau) & \cdots & q(i+n-\tau) \\ \vdots & \vdots & \ddots & \vdots \\ q(i-(D-1)\tau) & q(i+1-(D-1)\tau) & \cdots & q(i+n-(D-1)\tau) \end{bmatrix} \quad (8)$$

where i is an integer and k is the expected number of steps. To extract the behavior of the time series in an efficient way, the optimal values of D and τ must be determined. For the time series tested in this article, these values were determined by utilizing the average mutual information methods [40] and false nearest neighbor (FNN) counting.

Second step: receiving the time series matrix as a direct input, and the feedback output from the previous steps as a feedback input to the structure of the adaptive neural-fuzzy inference system. The direct input to the structure of the improved ANFIS is a matrix with an embedded phase vector as a column and a chaotic time series as a row. The direct entry of the matrix is as follows:

$$U = [u^T(i).u^T(i+1).,,,.u^T(i+m)]^T \quad (9)$$

The dimensions of the direct input matrix are $(m \times 1) \times D$, and i and k are the number of predicted steps.

The feedback input at time t of order M is as follows:

$$Y(i) = [y(t-1).y(t-2).,,,.y(t-N)] \quad (10)$$

The dimensions of the feedback input matrix are $(m+1)\times N$.

Third Step: Training the Improved Adaptive Neural-Fuzzy Inference System

The third step involves training the enhanced adaptive neural-fuzzy inference system using output feedback from the preceding step. This iterative process utilizes a hybrid algorithm combining the Imperialist Competitive Algorithm (ICA) and LSE. The objective is to iteratively refine the parameters of the system's front end, thereby enhancing prediction accuracy. The training process halts upon reaching the desired number of steps or achieving the target training error.[41]

In this procedure, the outputs of each stage "M" and their corresponding outputs from the previous stage serve as feedback inputs to the ANFIS. The training data, verification data, the specified number of training steps, and the type of membership functions for the fuzzy inference system are considered in determining optimal values for the front part parameters at each step using ICA. Subsequently, the parameters of the result part of the adaptive neural-fuzzy inference system are refined through LSE.

4. Experiments

In order to evaluate the suggested technique, comprehensive tests and simulations were conducted with the self-feedback ANFIS trained with the proposed learning algorithm. In all cases, 50% of the dataset is used for training, 20% for checking, and 30% for testing the model using the MATLAB software. The following criteria are employed to evaluate the numerical accuracy of the prediction.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(P_i - O_i)}{N}} \quad (11)$$

$$EF = 1 - \frac{\sum_{i=1}^{N}(P_i - O_i)^2}{\sum_{i=1}^{N}(O_i - \bar{O})^2} \quad (12)$$

$$VE = 1 - \frac{\sum_{i=1}^{N}\left(\left|\frac{P_i - O_i}{O_i}\right|\right)}{N} \times 100 \quad (13)$$

$$R^2 = 1 - \frac{\sum_{i=1}^{N}[(P_i - \bar{P})(P_i - \bar{O})]^2}{\sum_{i=1}^{N}(P_i - \bar{P})^2 \sum_{i=1}^{N}(O_i - \bar{O})^2} \quad (14)$$

where RMSE is the root mean square error, EF is the efficiency of the model, VE is the volumetric error (%), R is the explanatory factor, Pi is the predicted values by the model, Oi is the observed values, N is the number of data, P is the average predicted values by the model, and O is the average observed values. Therefore, any model with lower values than RMSE and VE values, and EF (the range of negative changes from infinity to one) and R closer to one will be more accurate than other models.

In evaluating the performance of the proposed method, several key considerations were taken into account to ensure a comprehensive and rigorous assessment. Performance metrics such as prediction total error criterion were employed to quantitatively measure the accuracy and effectiveness of the proposed method in forecasting chaotic time series data. The evaluation utilized multiple datasets, including real-world scenarios, to assess the robustness and generalizability of the proposed approach across different domains and data characteristics. Comparative analyses were conducted with recent works to benchmark the performance of the proposed method against established forecasting techniques, utilizing metrics such as forecasting accuracy and error rates. It's important to note that while the chosen evaluation methodology provides valuable insights into the performance of the proposed method, there may be inherent limitations and considerations to be aware of, such as dataset characteristics, model assumptions, and experimental conditions. Despite these considerations, the evaluation approach employed in this study aims to provide a transparent and comprehensive assessment of performance, contributing to the credibility and relevance of the research findings.

Firstly, data from each modality were collected and preprocessed individually to ensure consistency and compatibility. Preprocessing steps included data cleaning, normalization, and feature extraction, tailored to the characteristics of each modality. Subsequently, the preprocessed data were merged to create a unified dataset, utilizing appropriate techniques such as concatenation or feature fusion. Special consideration was given to ensure that the integrated dataset preserved the essential information from each modality while minimizing redundancy and maintaining data integrity. Furthermore, the rationale behind the chosen integration approach and any challenges encountered during the process are discussed to provide transparency and insight into the data integration methodology.

4.1.Mackey-Glass equation

The Mackey-Glass time series is provided with the time delay differential Eq. (15) and a model for the production of white blood cells. The mathematical form of this series is demonstrated in the following relation.

$$\frac{dx}{dt} = \frac{\alpha x(t-\lambda)}{1+x^{10}(t-\lambda)} - 0{,}1x(t) \qquad (15)$$

This time series is susceptible to initial conditions, and its behavior is chaotic for $\lambda \geq 16.8$ and has no clearly defined period. This time series has been used in many neural and fuzzy network modeling research studies. Figure 5 shows the response of this system for some different conditions. The standard values for reconstructing the state space of this time series are obtained by mutual information average and FNN methods. Therefore, the phase space of this series is reconstructed as $x(t\text{-}16), x(t\text{-}14), x(t\text{-}8), x(t)$ to predict $x(t\text{-}8)$ (eight steps ahead prediction). The dataset is partitioned into three subsets: 60% for training, 30% for validation, and 10% for testing. This division allows for effective training of the model on the majority of the data, while also enabling validation to ensure that the model generalizes well to unseen data. The testing subset serves as an independent evaluation set to assess the performance of the trained model on completely new data, providing an unbiased estimate of its effectiveness. This approach helps prevent overfitting and ensures that the model's performance is robust and reliable across different datasets.

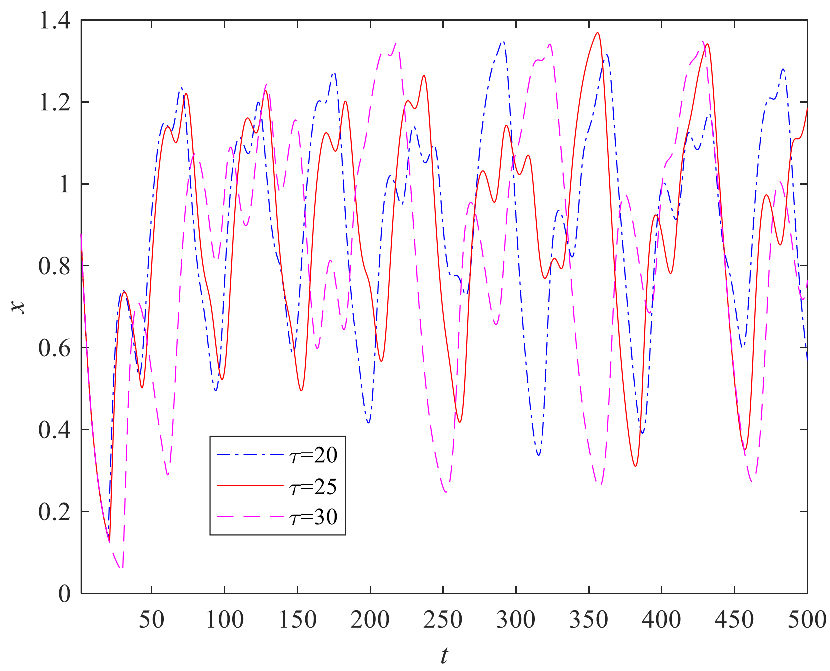

Fig. 5 Mackey-Glass time series response for different τ parameter

The suggested approach for predicting this time series is trained with the parameters mentioned in Table (1). The convergence and error reduction diagram of the suggested method is shown in Figure (6). The real and predicted values of the test time series are given in Figure (7), and the prediction error diagram is in Figure (8).

Table1. Parameters of the proposed method

Parameter	Value
Number of feedback inputs (N)	2
Clusters	40

Countries	50
Empires	10
Steps	200
α	2
β	1
Possibility of revolution	0.2
μ	0.03
ξ	0.02

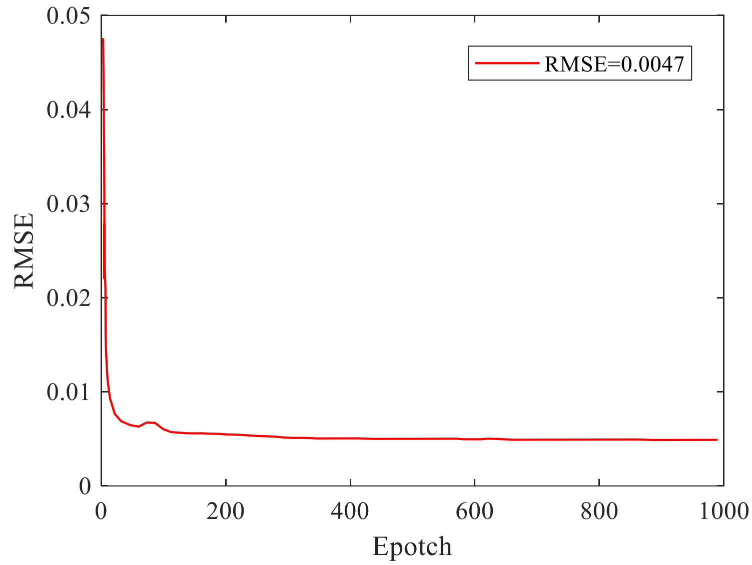

Fig. 6 Convergence of Mackey-Glass equation

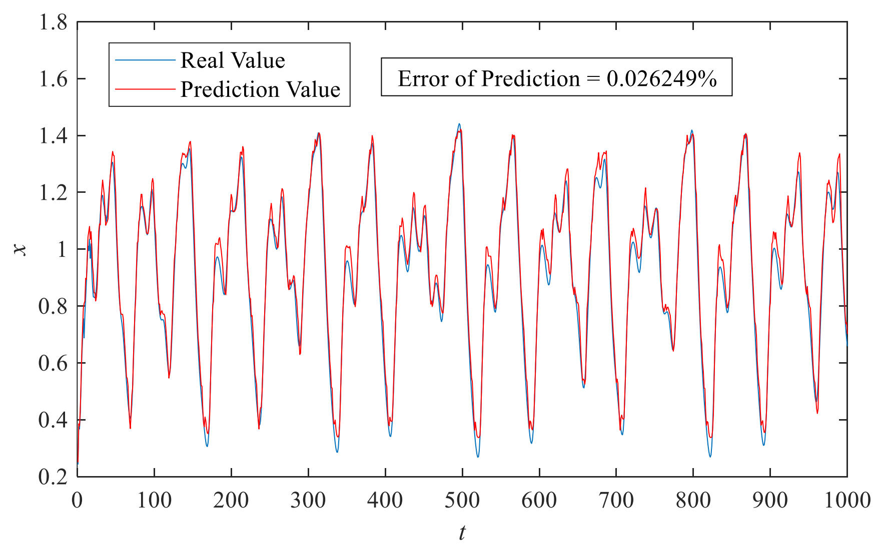

Fig. 7 Target and forecast values of Mackey-Glass time series

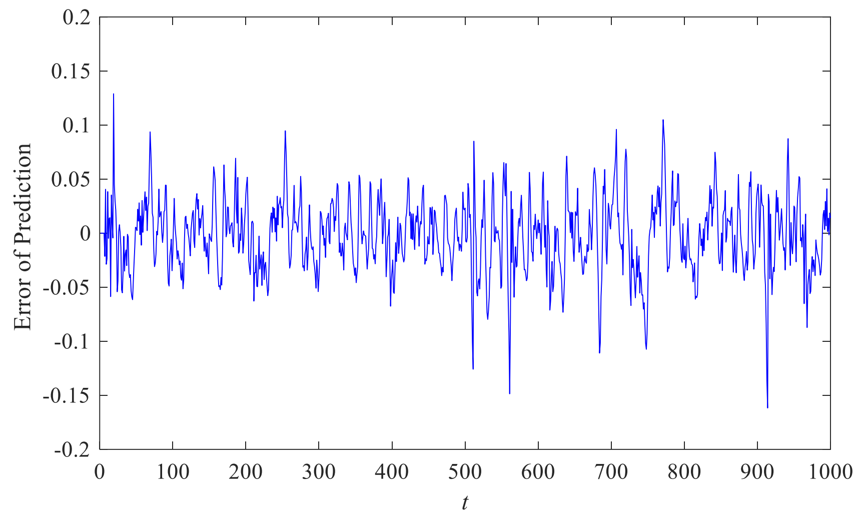

Fig. 8. Prediction error graph for Mackey-Glass equation

In order to numerically evaluate this method with previous methods for prediction, the RMSE of training and testing the suggested technique and some of the approaches presented in the articles are shown in Table (2). The last line of this Table shows the results of the proposed method. The results display the superiority and better efficiency of the presented method for forecasting this chaotic time series.

Table 2. The efficiency of Mackey-Glass time series prediction

Ref.	Technique	RMSE Train	RMSE Test
Chen et al. [42]	Auto-regressive model	-	0.19
Chen et al. [42]	Cascade correlation NN	-	0.06
Chen et al. [42]	Backpropagation NN	-	0.02
Chen et al. [42]	Linear prediction method	-	0.55
Lin et al. [29]	Product T-norm	-	0.09
Lin et al. [29]	Classical RBF (with 23 neurons)	-	0.0114
Lin et al. [29]	PG-RBF network	-	0.0028
Kim et al. [43]	Genetic algorithm and fuzzy system	-	0.049
Chen et al. [42]	LLWNN+gradient	0.0038	0.0041
Chen et al. [42]	LLWNN+hybrid	0.0033	0.0036
Yousefi et al. [44]	LLNF	0.0013	0.0020
Miranian et al. [45]	LNF with LSSVMs	0.00070	0.00079
	Proposed method	0.00015	0.00026

4.2. Lorenz time series

Lorenz time series is produced by three differential equations provided in Eq. (16).

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = \beta x - zx - y \\ \dot{z} = xy - \gamma z \end{cases} \quad (16)$$

In α = 10, γ = 3/8, β > 24.74, the system's output is chaotic. The standard values for reconstructing the state space of this time series are τ = 3, and D = 3. In the simulation to compare the time series with other similar works, the time series α = 10, β = 28, γ = 3.8 is considered. The input variables are x(t - 6), x(t - 3), x(t) to predict x(t - 3) (three step ahead forecast). This time series is proficient with the parameters mentioned in Table (1). The convergence figure and error reduction of the suggested method are shown in Figure 9.

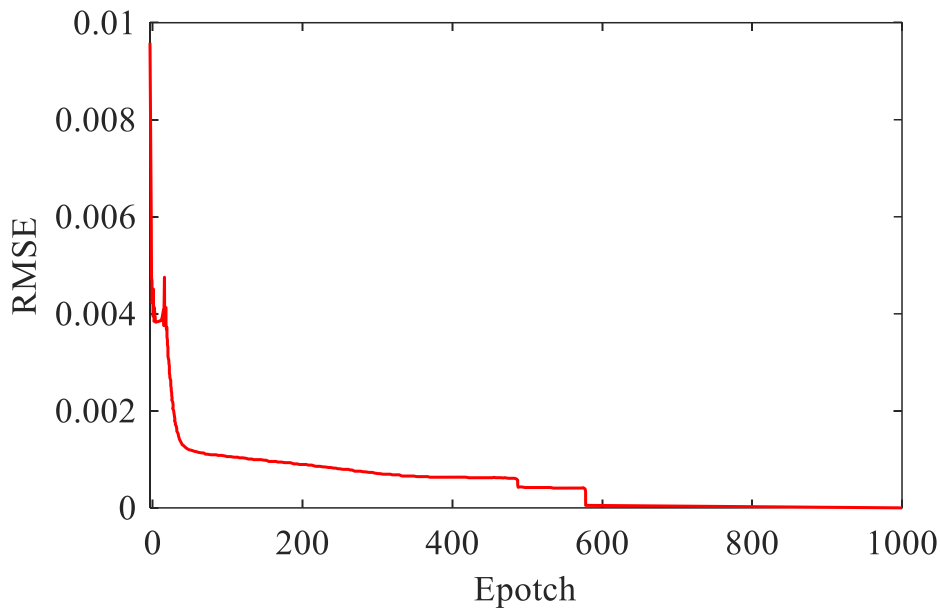

Fig. 9 Convergence of RMSE diagram for Lorenz time series

The predicted output by the current research method and the prediction error diagram are displayed in Figures 10-11.

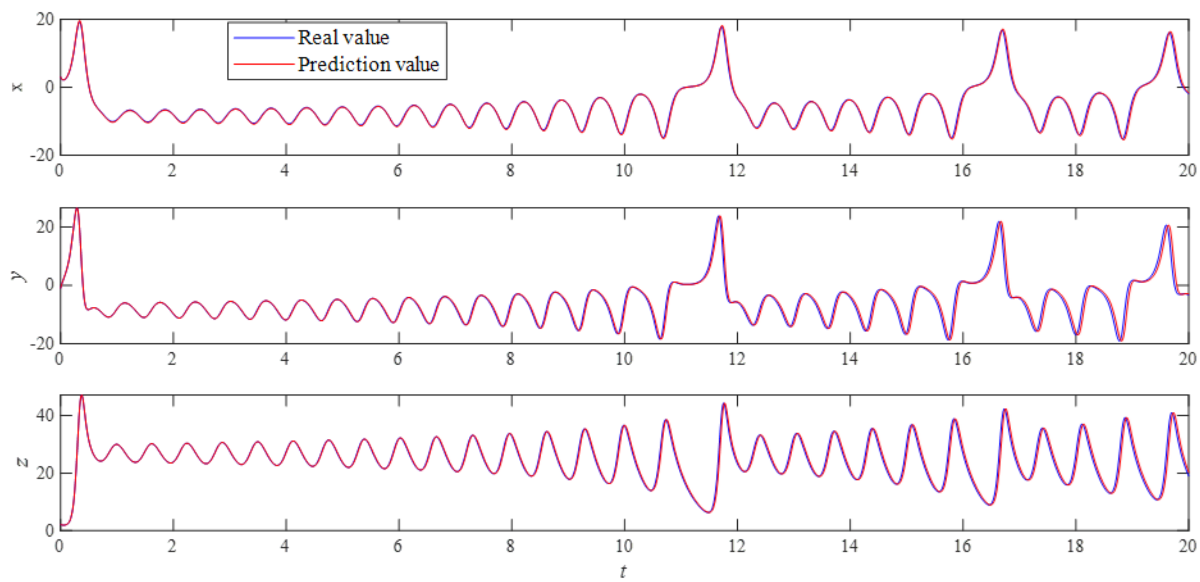

Fig. 10 Forecast output graph for Lorenz time series

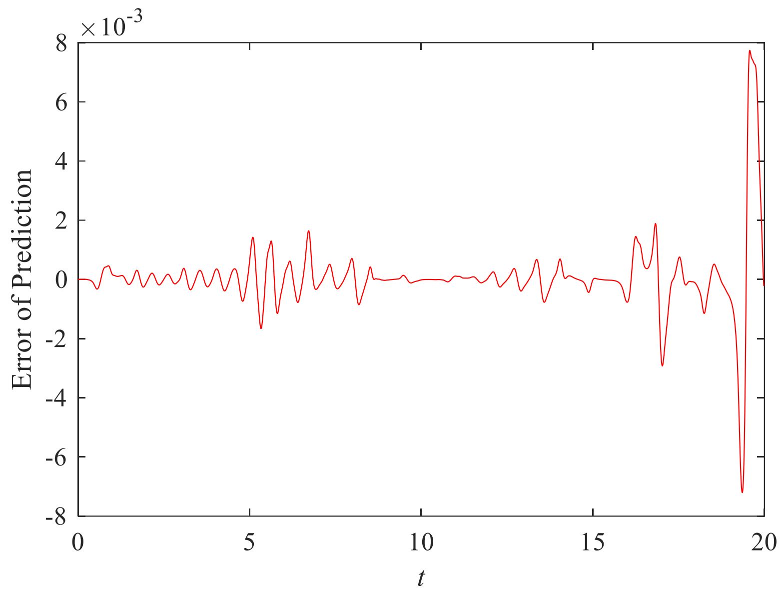

Fig 11. Prediction error graph for Lorenz time series

In order to numerically evaluate this method with previous prediction methods, the NMSE of training and testing of the suggested method and a number of approaches presented in the articles are shown in Table (3). The last line of this table shows the results of the suggested method. The results demonstration the superiority and better efficiency of the suggested technique for forecasting this chaotic time series.

Table 3. Comparison of forecast efficiency of Lorenz time series

Ref.	Method	RMSE Train	RMSE Test
Mirikitani et al. [46]	MLP-EKF	0.00023	0.00162
Mirikitani et al. [46]	MLP-BLM	0.00033	0.00096
Mirikitani et al. [46]	RNN-BPTT	0.00056	0.00185
Chen et al. [42]	ANFIS	0.0026	0.0021
Lin et al. [29]	Fuzzy prediction based on SVD	-	0.0106
Lin et al. [29]	LLNF	0.00013	0.00029
Miranian et al. [45]	LNF with LSSVMs	-	0.000064
Chen et al. [42]	ICA-ANFIS	0.00007	0.00012
	Proposed method	0.0000051	0.000056

4.3. Rossler time series

The Rossler model is characterized by a system of three ordinary differential equations, defining a continuous dynamic system. This system represents the chaotic dynamics associated with the fractal properties of Rossler absorption. The following differential equations define the Rossler time series:

$$\begin{cases} \dot{x} = -z - y \\ \dot{y} = -ay + x \\ \dot{z} = b + z(x - c) \end{cases} \tag{17}$$

For a = 0.2, b = 0.2, and c = 4.6, the system's behavior becomes chaotic. The standard values for reconstructing the state space of this time series are $\tau = 2$ and $D = 3$. The input variables are x(t-4), x(t-2), x(t) to predict x(t-2) (prediction two steps ahead). This time series is accomplished with the parameters mentioned in Table (1). The predicted output is shown in Figure 12. As can be seen, using the presented method, the results of this nonlinear equation are also predicted with a very favorable accuracy.

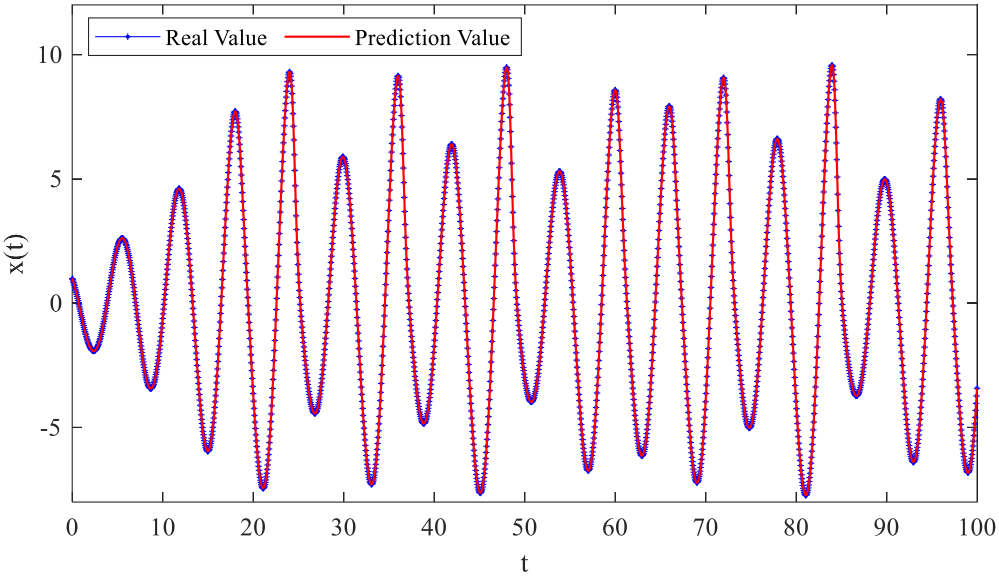

Fig. 12 Forecast output graph for Rossler time series

In order to numerically evaluate this method with previous prediction methods, NMSE of training and testing of the suggested method and some of the methods presented in the articles are offered in Table (4). The last line of this Table shows the results of the proposed method. The results show the superiority and better efficiency of the proposed method for forecasting this chaotic time series.

Table 4. Comparison of Rossler's time series prediction efficiency

Ref.	Method	RMSE Train	RMSE Test
Mirikitani et al. [46]	MLP-EKF	0.00025	0.00193
Mirikitani et al. [46]	MLP-BLM	0.00101	0.00047
Mirikitani et al. [46]	RNN-BPTT	0.00311	0.00070
Jang [39]	ANFIS	0.0118	0.0147
Yousefi et al. [44]	LLNF	0.000071	0.000048
Miranian et al. [45]	LNF with LSSVMs	0.000015	0.0000065
	Proposed method	0.000004	0.0000007

4.4. *Sunspot Number time series*

The time series of the number of sunspots is an unstable and very complicated time series of the real world. This series is related to the annual average number of observed sunspots. The average time series of sunspots recorded from 1700 to 1979 is presented in [66]. The standard values for reconstructing the state space of this time series are $\tau = 1$ and $D = 4$. Therefore, the input variables are x(t - 4), x(t - 3), x(t -2), x(t -1), x(t) to predict x(t +2) (prediction two step ahead). The convergence diagram and error reduction of the suggested method are depicted in Figure 13.

The real and forecast values of the test time series are displayed in Figure 14. The results show the high accuracy of the presented method in predicting the temporal behavior of nonlinear dynamic systems.

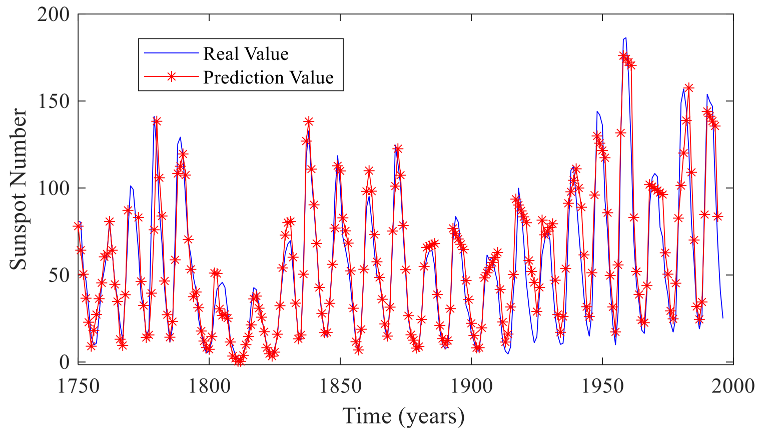

Fig. 14 Prediction output graph for Sunspot Number time series

In order to numerically evaluate this method with previous prediction methods, the RMSE of training and testing of the proposed method and several methods presented in the articles are provided in Table (5). The last line of this Table shows the results of the proposed method. The results show the superiority and better efficiency of the presented method for forecasting this time series.

Table 5. Sunspot Number time series prediction efficiency

Ref.	Method	RMSE Train	RMSE Test
Tong et al. [47]	Threshold autoregression	0.097	0.96
Weigend et al. [48]	Statistical method	0.082	0.086
Svarer et al. [49]	AFGF	0.090	0.082
Aliev et al. [50]	RFNN	-	0.074
Miranian et al. [45]	LNF with LSSVMs	0.050	0.063
Zhang et al. [51]	FWNN-M	0.0828	0.0973
Proposed method	Proposed method	0.0152	0.0265

In continuation of the present work, future studies could explore several avenues to further enhance the proposed method's effectiveness in dynamic time series prediction. These include fine-tuning model parameters to optimize predictive accuracy, extending the framework to handle multivariate time series data and diverse application domains, incorporating external factors or exogenous variables into the modeling process, evaluating the method's performance for long-term forecasting horizons, benchmarking against state-of-the-art techniques, exploring hybrid approaches to leverage complementary strengths, and developing strategies for real-time implementation and deployment. By addressing these areas, researchers can advance the understanding and applicability of the enhanced adaptive neural-fuzzy inference system, contributing to advancements in time series forecasting and predictive analytics.

5. Conclusion

This paper presents an enhanced version of the adaptive neuro-fuzzy inference system (ANFIS) with output feedback, trained using the Imperial competitive algorithm (ICA) for forecasting chaotic time series. By incorporating output feedback from previous stages, the ANFIS model overcomes its static nature and enhances its ability to handle dynamic issues and temporal changes in the data. Furthermore, a combination of ICA and least squares estimation (LSE) is utilized to optimize the model parameters during training. This learning algorithm makes it possible to identify and adjust the optimal structure of the adaptive neural-fuzzy inference system in the most desirable way while eliminating the problems caused by updating the parameters with gradient-based methods. In addition, the complexity of this algorithm is less compared to gradient-based methods. The results of using this method to predict chaotic time series show a remarkable improvement in the performance of the improved adaptive neuro-fuzzy inference system compared to previous methods. This shows the proper performance of the improved adaptive neural-fuzzy inference system in dealing with dynamic problems and its incredible ability to learn and train parameters.

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